CALCULATION OF FLUCTUATION CHARACTERISTICS OF TURBULENT FLOWS

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In describing the behavior of a continuous medium one uses invariant tensors, characterizing the state of the medium [1-4]. The ideas forming the basis of determining the relation between the invariants in the mechanics of a solid deformed body [2, 4] can also be used in fluid mechanics.

Invariants of the Reynolds Stress Tensor. The Reynolds stress tensor $\mathbf{R}$ can be written down in componentless (coordinateless) form [1]

$$\mathbf{R} = -\rho\overline{\mathbf{v}' \otimes \mathbf{v}'}.$$

Here $\mathbf{v}'$ denotes the fluctuating velocity vector, ρ is the fluid density, $\otimes$ is the symbol of tensor multiplication, and the bar is Reynolds averaging. This tensor, as any other second-rank tensor, can be represented in an arbitrary basis $\mathbf{e}_i$ as $\mathbf{R} = R^{ij}\mathbf{e}_i \otimes \mathbf{e}_j$. Its physical components R_{ij} form a matrix, whose elements are well described by measurements in the flow by means of a thermoanemometer. In the basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ with corresponding components (u, v, w) of the velocity vector $\mathbf{v}$ this matrix is

$$\begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} = \begin{pmatrix} -\rho\overline{u'^2}; & -\rho\overline{u'v'}; & -\rho\overline{u'w'} \\ -\rho\overline{u'v'}; & -\rho\overline{v'^2}; & -\rho\overline{v'w'} \\ -\rho\overline{u'w'}; & -\rho\overline{v'w'}; & -\rho\overline{w'^2} \end{pmatrix} \tag{1}$$

(the primes refer to the fluctuating quantities). The experimental data show that the symmetric tensor $\mathbf{R}$ has a large spherical part and a relatively small variance portion.

For any second rank tensor, including $\mathbf{R}$, one can write a characteristic equation

$$\sigma^3 - \mathrm{I}\sigma^2 + \mathrm{II}\sigma - \mathrm{III} = 0, \tag{2}$$

where σ are the principal (eigen) values of the tensor, and I, II, III are its invariants, determined by the equations

$$\mathrm{I} = \operatorname{tr}\mathbf{R},\quad \mathrm{II} = \frac{1}{2}((\operatorname{tr}\mathbf{R})^2 - \operatorname{tr}\mathbf{R}^2),\quad \mathrm{III} = \det\mathbf{R} \tag{3}$$

(tr and det are the trace and determinant, respectively). Knowing the matrix (1), from Eqs. (3) one can calculate the invariants I, II, III. In particular, for the first invariant

$$\mathrm{I} = \operatorname{tr}\mathbf{R} = -\rho(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) = -\rho k \tag{4}$$

(k is twice the kinetic energy of fluctuations). It is hence seen that the first invariant, characterizing the mean normal stress at the point under consideration, is proportional to the kinetic energy of turbulent fluctuations. Similarly, one can determine from Eqs. (3) the invariants II and III.

The reality of roots of the cubic equation (2) imposes certain restrictions on the relations between its coefficients. Using this fact, based on the presently available vast experimental material, one can attempt to find empirically the relation between the invariants of the Reynolds stress tensor. Figure 1 shows the results of processing a large number of various experimental data [5-12]. Here 1 are the experimental points corresponding to a planar channel [5], 2 is a circular tube [6], 3 is a boundary layer on a plate [7], 4 is planar Couette flow [8], 5 is pressurized Couette flow (experiment 3 [8]), 6 is pressurized Couette flow (experiment 13 [8]), 7 is a planar jet [9], 8 is a circular jet [10], 9 is a rotating cone [11], and 10 is a rotating plane [12]. It is seen that among the invariants II and I, III and I there exists a relation, expressed as a dependence of the second and third invariants on the first, proportional to the fluctuation kinetic energy. However, despite the small interval of invariants $0.22 \leq \mathrm{II}/\mathrm{I}^2 \leq 0.33$; $0.010 \leq \mathrm{III}/\mathrm{I}^3 \leq 0.025$, the

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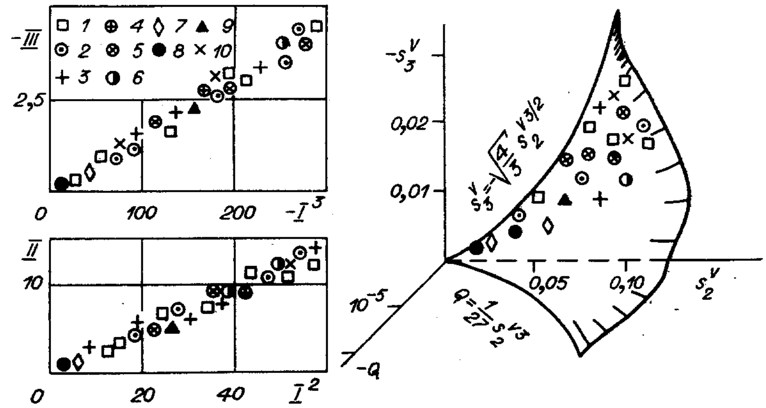

Fig. 1

stress tensor and the corresponding stress state at the flow point under consideration vary strongly with variations in these quantities. This is explained by the fact that on the background of the predominant spherical portion of the tensor **R** the available characteristics of the stress state, determined by the second and third invariants, become poorly distinguished. Therefore, along with invariants II and III one must relate the second and third invariants of the variance of the tensor $\mathbf{R} - (1/3)(\mathrm{tr}\,\mathbf{R})\mathbf{G}$. In the studies of Lumley, Mathieu, and Ghandel [13] is analyzed the effect of these invariants on the characteristics of turbulent flows. However, if one constructs the dependences of the variance invariants on the first invariant, it can be verified that they have the nature of a "starry sky," as a result of which is imposed an impression of the impossibility of searching any relations between the invariants. To clarify the problem of relation between the invariants it is convenient to select the second and third invariants of the variance part in the form suggested in [2, 4]. These invariants are related to the coefficients of the cubic equation correspondong to (2).

The cubic equation (2), having three real roots σ_1, σ_2, σ_3, can be represented as follows:

$$(\sigma/\mathrm{I})^3 - (\sigma/\mathrm{I})^2 + (\mathrm{II}/\mathrm{I}^2)(\sigma/\mathrm{I}) - \mathrm{III}/\mathrm{I}^3 = 0.$$

Substituting $(\sigma/\mathrm{I}) = z - 1/3$, it is reduced to incomplete form

$$z^3 + pz + q = 0,\ p = \mathrm{II}/\mathrm{I}^2 - 1/3,\ q = -2/27 + (1/3)\mathrm{II}/\mathrm{I}^2 - \mathrm{III}/\mathrm{I}^3. \tag{5}$$

Since the roots of the cubic equation for **R** are real (there can be no imaginary fluctuations), then $p \leqslant 0$, $Q \leqslant 0$ [14], where

$$Q = (p/3)^3 + (q/2)^2. \tag{6}$$

The second and third invariants of the variance of the stress tensor are expressed in terms of the coefficients p and q:

$$s_2^V = -p = 1/3 - \mathrm{II}/\mathrm{I}^2;\quad s_3^V = 3q = -2/9 + \mathrm{II}/\mathrm{I}^2 - 3\mathrm{III}/\mathrm{I}^3, \tag{7}$$

while $0 \leqslant s_2^V \leqslant 1/3$, $s_3^V < 0$. The physical meaning of the second invariant s_2^V is the ratio of the mean tangential stress to the mean normal [4]. Along with s_3^V it is convenient to choose as third invariant the quantity ξ, the shape angle, determined according to [4] by the relation

$$\sin 3\xi = s_3^V / \sqrt{(4/3)(s_2^V)^3};\quad -\pi/6 \leqslant \xi \leqslant \pi/6. \tag{8}$$

The shape angle ξ characterizes the ratio of the mean tangential stress at the point under consideration to the maximum tangential stress.

Relations between Invariants. Thus, if ρk, s_2^V, ξ are selected as system invariants of the Reynolds stress tensor, the principal normal stresses σ_1, σ_2, σ_3 (with $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$) can be written as [2]

$$\begin{aligned} \sigma_1 &= -\rho k\left(1/3 + 2\sqrt{s_2^V/3}\sin(\xi + 2\pi/3)\right), \\ \sigma_2 &= -\rho k\left(1/3 + 2\sqrt{s_2^V/3}\sin\xi\right), \\ \sigma_3 &= -\rho k\left(1/3 + 2\sqrt{s_2^V/3}\sin(\xi + 4\pi/3)\right). \end{aligned} \tag{9}$$

To find relations between the invariants $\rho k, s_2^V, \xi$ it is useful to turn attention to the function Q, determined by expression (6). Taking into account (7), we have $Q = -(s_2^V/3)^3 + (s_3^V/6)^2$. For Q = 0 it follows from the last expression that the following relation holds between the invariants of the tensor **R**:

$$s_3^V = -\sqrt{4/3}\,(s_2^V)^{3/2}. \tag{10}$$

Substitution of (10) into (8) gives $\sin 3\xi = -1$, whence $\xi = -\pi/6$. In this case, as is seen from (9), if $0 < s_2^V < 1/3$, the principal stresses are $\sigma_2 = \sigma_3$, and the ellipsoid tensor for **R** is an ellipsoid of revolution. The limiting cases $s_2^V = 0$ and $s_2^V = 1/3$ deserve separate consideration. If $s_2^V = 0$, all three roots of the characteristic equation are identical [$\sigma_1 = \sigma_2 = \sigma_3 = -(1/3)\rho k$], all fluctuations are identical, and turbulence is isotropic (the tensor ellipsoid transforms in this case to a sphere). For $s_2^V = 1/3$ $\sigma_2 = \sigma_3 = 0$ we also have the situation in which the velocity fluctuations are lumped in a single direction, corresponding to σ_1 (the ellipsoid tensor transforms into a straight line). This corresponds to maximum anisotropy of fluctuations. The problem provided above shows that the invariant s_2^V can be assigned in one sense – in a turbulent flow it characterizes the anisotropy of fluctuations.

The case Q = 0 corresponds to vanishing differences of normal stresses, i.e., vanishing tangential stresses for areas whose orientation is related to the flow direction. If $\sigma_1 \neq \sigma_2 \neq \sigma_3$, then $Q \neq 0$, and the relation between the invariants of the tensor **R** becomes more complex according to Eq. (10) and the subsequent $\xi = -\pi/6$ following from it. Constructing the dependence of Q on s_2^V and s_3^V from experimental data for diverse flows [5-12], the experimental points are located on some surface (see Fig. 1), while for Q = 0 they fall on a curve described by (10). Imposing all these experimental points on a single curve [projecting the surface on a line in the coordinates $(\rho k, s_2^V, \xi)$] is possible only by including the deformation flow characteristics determined by the tensor of averaged deformation velocities. The situation here is similar to mechanics of a deformed solid, where, as well known [15], the invariants of the stress tensor are related by means of the invariants of the deformation tensor.

Analysis of the distribution of points on the surface shown in Fig. 1 leads to the empirical equation

$$Q = -(1/3)\left((4/3)\,\tau/\rho k\right)^2 (s_2^V)^2, \tag{11}$$

where τ is an invariant scalar quantity, characterizing the ratio of the specific power of Reynolds stresses to the shear intensity:

$$\tau = |\mathbf{R} : \mathbf{d}| / \sqrt{2|\mathbf{d} : \mathbf{d}|} \tag{12}$$

(**d** is the averaged tensor of deformation velocities, and the double point denotes the double scalar product of tensors). For a simple shear flow it follows from (12) that τ is simply the modulus of the tangential stress, $\overline{\rho u'v'} \equiv R_{12}$.

Using relations (6) and (8), together with (11) one can write [16]

$$\cos 3\xi = 4\tau/\rho k. \tag{13}$$

In Fig. 2 (the notation is the same as in Fig. 1) we provide the experimental curve in the coordinates $\tau/\rho k - \cos 3\xi$, whose approximation is also the empirical equation (13), showing that the ratio $\tau/\rho k$ varies from 0 to 0.25. It is interesting to note that the numerous experimental data (flows in boundary layers, jets, wakes) imply [17] constancy of $\tau/\rho k$ in a large portion of cross sections of these flows. This served as justification for using in the differential transport models of the kinetic energy [18] the relation $\tau/\rho k = \text{const} = a_1$, called in the literature the Nevzglyadov – Dryden relation (while Bredshaw and Ferris use $a_1 = 0.15$). Account of the nonconstancy of the ratio $\tau/\rho k$, according to Eq. (13), can substantially refine the calculation of fluctuations, particularly near the boundaries of turbulent flows.

Also shown in Fig. 2 is the agreement with experiments of the first (second) empirical relation, relating the three invariants of the Reynolds stress tensor:

$$20 s_2^V \left(\cos 3\xi + 3V_0/\sqrt{k}\right) = 1, \tag{14}$$

where V_0 is determined by the expression

$$V_0 = |\nabla\tau| / 2\rho\, |II_d|^{1/2}; \tag{15}$$

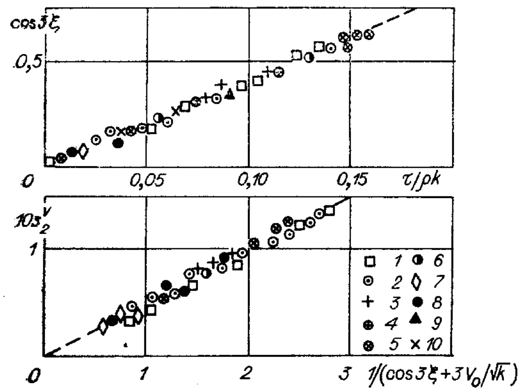

Fig. 2

∇ is the gradient operator, and II_d is the second invariant of the velocity deformation tensor $\mathbf{d}$. For simplicity of shear flows V_0 is determined by derivatives with respect to the transverse coordinate: $V_0 = \left|\frac{d\tau}{dy}\right| \Big/ \rho \left|\frac{du}{dy}\right|$. The empirical equations (13) and (14) can be used to calculate the establishment of turbulent flows of an incompressible fluid.

Governing Equation for the Reynolds Stress Tensor. In solving problems of establishing isothermal turbulent flows of an incompressible fluid, it is necessary to close the system of equations consisting of the equations of motion and continuity. For this it is necessary to write down the governing (rheological) relation for $\mathbf{R}$, relating it to the average tensor of deformation velocities $\mathbf{d}$, while the relation must account for nonlinearity, anisotropy at the off-axis level of $\mathbf{R}$ and $\mathbf{d}$, and the memory of the turbulent flow. As implied by experiments of related branches in mechanics of continuous media (theory of plasticity with anisotropic hardening [3], encountering similar problems), the governing relation accounting for the complex effects enumerated above necessarily has a differential shape. At the same time one widely uses in turbulence theory differential transport models, constructed on the basis of the known transport equations of Reynolds stresses [19], written down by Reynolds himself. These equations contain a number of correlation terms, requiring some approximation.

The presently available differential models for the transport tensor $\mathbf{R}$ are based on the isotropic approximation of the terms indicated above, each such term being approximated separately, independently of the others. In this case the purpose does not become an explicit correct description (according with experiment) of the simultaneous axes of the tensors $\mathbf{R}$ and $\mathbf{d}$, as well as the necessity of transforming the model in the case of simple shear flows to the well-recommended Boussinesq equation $\tau_{12} = \mu_t du/dy$ (μ_t is the turbulent viscosity). As a result the suggested turbulence models cover only part of the class of problems, including those which are solved with sufficient accuracy by much simpler methods. In attempts of extending the possibilities of the transport equation, they contain a large number of auxiliary functions and empirical constants. In several variants of the theory the number of the latter reaches 20, and the theory acquires an explicitly interpolation character: the number of constants practically does not exceed the number of problems for which the theory gives satisfactory results [19].

In constructing the differential governing relation at a phenomenological level it is natural to require an approximation not of the separate terms contained in the Reynolds transport equation for $\mathbf{R}$, but of the whole effect of these terms in the complex, so as to describe correctly (according to experimental data) the anisotropy, the nonaxial feature of $\mathbf{R}$ and $\mathbf{d}$. Besides, the rationally constructed differential transport model for $\mathbf{R}$ must lead in the limiting case of simple shear flows to algebraic models of the Boussinesq type, while the flow curvature is accounted for in this case automatically, without introducing any corrections. In writing down the differential governing relation it is necessary to follow, based on experiments, the purpose of the simplest economical description (from the point of view of number of phenomenological constants, as well as the algorithm of flow calculations) of nonlinearity, anisotropy, and memory of turbulent flows. The simplicity of the governing relation is the foundation of its successful use in engineering applications.

Starting from these considerations, to establish the flow of an incompressible fluid one can assume [16] a governing relation for $\mathbf{R}$, written in componentless form:

$$(\mathbf{v}\cdot\nabla)\mathbf{R} = -A\tau\mathbf{d} - 2\mu_t(\mathbf{d}\cdot\mathbf{w} + (\mathbf{d}\cdot\mathbf{w})^{\mathrm{T}}) + (\mathbf{R}\cdot\mathbf{w} + (\mathbf{R}\cdot\mathbf{w})^{\mathrm{T}}), \tag{16}$$

where $\mathbf{d}$ and $\mathbf{w}$ are the tensors of deformation velocity and of the average rotation ($\mathbf{d} = (1/2) \times (\nabla\mathbf{v} + (\nabla\mathbf{v})^{\mathrm{T}})$, $\mathbf{w} = (1/2)(\nabla\mathbf{v} - (\nabla\mathbf{v})^{\mathrm{T}})$); $\mathbf{v}$ is the average velocity vector, τ is defined by expression (12), the symbol T denotes transposition, and μ_t is the turbulent viscosity which, as well as the other invariant scalar quantity A, having the meaning of dimensionless diffusion coefficient for $\mathbf{R}$, is determined at a phenomenological level.

In component form in a Cartesian rectangular basis Eq. (16) acquires the form

$$v_k\frac{\partial R_{ij}}{\partial x_k} = -A\tau d_{ij} - 2\mu_t(d_{ik}w_{kj} + d_{jk}w_{ki}) + (R_{ik}w_{kj} + R_{jk}w_{ki}),$$
$$d_{ij} = \frac{1}{2}\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right), \quad w_{ij} = \frac{1}{2}\left(\frac{\partial v_j}{\partial x_i} - \frac{\partial v_i}{\partial x_j}\right). \tag{16a}$$

Based on the analysis of experimental data [5-12]

$$A\tau = \frac{7\tau}{\sqrt{1+2(\mu_t^V)^{1/8}}} + \frac{1}{2}\rho V_0^2, \tag{17}$$

where τ and V_0 are defined by expressions (12) and (15), and $\mu_t^V = \mu_t/\mu$ (μ is the dynamic viscosity). The scalar μ_t in (16) and (17) must also be written down in invariant form. It is conveniently selected within the generalized Van Karman theory, simply and effectively describing a variety of turbulent flows with the use of only two phenomenological constants [20-22]:

$$\mu_t = \rho\nu\varkappa_n T^n. \tag{18}$$

Here ν is the kinematic viscosity, n and $\varkappa_n$ are phenomenological constants, while for the "Blasius" [21] region of Reynolds numbers, most interesting for practical applications, n = 3/4, $\varkappa_n$ = 0.53 (this flow region in a tube occupies approximately two decades, Re ≈ 10^4-10^6), and T is the local Reynolds number in the Karman form:

$$T = \frac{|\,2\,|\,2\mathbf{d}:\,\mathbf{d}\,|^{1/2} - |\,2\mathbf{w}:\,\mathbf{w}\,|^{1/2}\,|^3}{\nu\,(2\nabla\,|\,\boldsymbol{\omega}\,|)^2} \tag{19}$$

[$\boldsymbol{\omega} = (1/2)(\nabla\times\mathbf{v})$ is the rotor of the average velocity]. In particular, for flow in a planar channel, a circular tube, and for circular Couette flow (for which $\omega = u/r$) this generalized T leads, respectively, to the expressions

$$T = \frac{|\,u'\,|^3}{\nu\,|\,u''\,|^2}, \quad T = \left|\frac{du}{dr}\right|^3\Big/\nu\left|\frac{d^2u}{dr^2}\right|^2, \quad T = \frac{|\,2\,|\,r\omega''\,| - |\,2\omega + r\omega'\,|\,|^3}{\nu\,|\,3\omega' + r\omega''\,|^2} \tag{20}$$

(the primes denote derivatives with respect to the transverse coordinate).

The model (16) is primarily the new governing relation for the Reynolds stress tensor in the form of a transport equation, simultaneously accounting for the nonlinearity, anisotropy, and memory of turbulent flows, which:

1) approximates the terms of the Reynolds equation for turbulent stresses not isotropically, as in the existing differential transport models, but anisotropically, by means of introducing into the treatment the averaged rotation tensor (by the antisymmetric part of the gradient tensor of average velocities $\nabla\mathbf{v}$, guaranteeing repetition of the principal axes of the deformation velocity tensor to the axes of the stress tensor);

2) leads for simple shear flows to algebraic governing relations, and includes the limiting special case of algebraic expressions for the turbulent tangential stress, while such expressions can also be the Prandtl model of mixing length, the turbulent viscosity model, and the V. V. Novozhilov equation according to the generalized Karman theory. It is noted that if for the expression for μ_t one uses relations (18), (19), the model contains a minimal amount of phenomenological constants, being universal for the "Blasius" region of Reynolds numbers;

3) automatically accounts for flow curvature, rendering unnecessary the introduction of empirical curvature corrections, characteristically the case for the currently available turbulence theories. The latter is easily verified, for example, by solving the problem of turbulent Couette flow between two rotating cylinders and comparing the solution with the known experimental data of Taylor and Wendt [16].

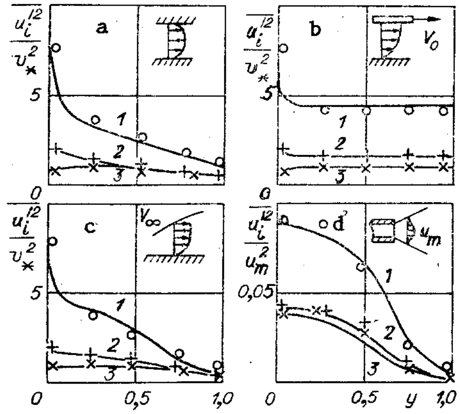

Fig. 3

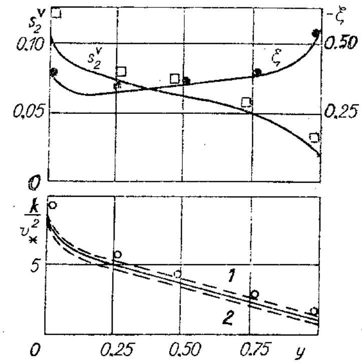

Fig. 4

Calculation of Fluctuation Characteristics of Turbulent Flows. The calculation scheme of normal Reynolds stresses (and consequently, fluctuating flow characteristics) for establishing simple shear flows and adjacent flows is most simply treated on the example of flow in a planar channel. Writing Eq. (16) in component form as applied to establishment of flow in a planar channel, where there exists only a longitudinal velocity component u(y), depending on the transverse coordinate, we then obtain

$$0 = A\tau \frac{du}{dy}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + 2\mu_t\left(\frac{du}{dy}\right)^2\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{du}{dy}\begin{pmatrix} 2R_{12}; & R_{22}-R_{11}; & 0 \\ R_{22}-R_{11}; & -2R_{12}; & 0 \\ 0; & 0; & 0 \end{pmatrix}$$

where A, τ, and μ_t are defined by relations (12), (17)-(19). As a result we have a system of two algebraic rheological relations (for tangential and normal stresses):

$$R_{12} \equiv \tau = \mu_t \frac{du}{dy}; \quad \mu_t = \rho\nu\varkappa_n T^{3/4},$$
$$R_{11} - R_{22} = -a; \quad a = \frac{7\tau}{\sqrt{1+2(\mu_t^{\vee})^{1/8}}} + \frac{1}{2}\rho V_0^2. \tag{21}$$

Similar relations can also be found for other simple shear turbulent flows and adjacent flows, admitting a simplification of boundary-layer type.

System (21) shows that the tangential and normal stresses of this flow are distinct. And since the normal stresses are determined by the tangential ones and their gradients, it is first necessary to perform a calculation of the tangential stress and the averaged velocity field on the basis of the first equation of system (21), and then calculate τ, μ_t, II_d, and V_0. In the present study this and all subsequent calculations were carried out by the generalized Karman theory [21]. Calculations can also be performed by any other semiempirical theory; they all give closely related results, at least for a simple flow such as a channel. Also, the final results of the following calculations of fluctuations are practically identical, independently of the scheme of calculating τ and the velocity profiles.

Before turning to the treatment of the second equation of system (21) it is useful to rewrite the equations known from elasticity theory [23] for a planar stress state (when $\sigma_{33} = \sigma_3$) as applied to the Reynolds stress tensor:

$$R_{11} = -\left(1/3 - \sqrt{s_2^{\vee}/3}\sin\xi + \sqrt{s_2^{\vee}}\cos\xi\cos 2\alpha\right)\rho k,$$
$$R_{22} = -\left(1/3 - \sqrt{s_2^{\vee}/3}\sin\xi - \sqrt{s_2^{\vee}}\cos\xi\cos 2\alpha\right)\rho k, \tag{22}$$
$$R_{33} = -\left(1/3 + 2\sqrt{s_2^{\vee}/3}\sin\xi\right)\rho k,$$

where α is the inclination angle of the principal axis of the stress tensor with respect to the flow, determined by the relation

$$\tau \equiv R_{12} = \sqrt{s_2^{\vee}}\cos\xi\sin 2\alpha\rho k. \tag{23}$$

Equations (22) and (23) are valid for any simple shear flow. The second equation of system (21) with account of (22) is reduced to the form

$$2\rho k\sqrt{s_2^{\vee}}\cos\xi\cos 2\alpha = a. \tag{24}$$

Equations (13), (14), and (24) form a system of three algebraic equations in the three unknown invariants ρk, $\overset{\vee}{s_2}$, ξ. Following its solution, one can calculate the normal stresses $R_{11} = -\rho\overline{u'^2}$, $R_{22} = -\rho\overline{v'^2}$, $R_{33} = -\rho\overline{w'^2}$ by Eqs. (22).

Figure 3 shows a comparison of calculations of the fluctuating characteristics of turbulent flows with experiment [5-10] for various flows: a) in a planar channel, Re = 10^5, b) planar nonpressurized Couette flow, Re = 10^5, c) in the boundary layer of a planar film, $Re_x = 10^6$, d) in a planar jet, $Re_b = 10^5$; the line is the calculation (1 is the longitudinal fluctuation component, 2 is the transverse, and 3 is the component across), and the points are experiment.

Figure 4 shows the behavior of the invariant of the Reynolds stress tensor across the flow for flow in a planar channel as a function of transverse coordinate measured from the wall; the solid lines are the calculation with Re = 10^5, the points are experiment [5], and the dashed lines 1 and 2 show the effect of the Reynolds number on the fluctuation kinetic energy for Re = 10^4 and 10^8.

For more complicated stationary turbulent flows of an incompressible fluid (spatially) it is necessary to solve a system of differential equations in partial derivatives, which is possible only by numerical methods. This system consists of the equations of motion of a continuous medium in the stresses (the momentum balance equations), continuity, and the governing equation (16). Following the statement of the corresponding boundary conditions, the problems of calculating the turbulent flow become boundary-value problems of mathematical physics. It is interesting to note that even in these complicated problems the structure of Eqs. (16) allows the system of equations, written in scalar form, to be partitioned into two groups, the first of which is independent of the second and provides the possibility of carrying out the calculation of tangential stresses and average velocities, while the second, following calculations on its basis, allows one to calculate the fluctuating flow characteristics.

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TWO-PHASE FILTRATION IN MIXED-WETTABLE POROUS MEDIA

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The pressure difference

$$p - p_1 = p_c, \tag{0.1}$$

called the "capillary pressure at the phase boundary," is the governing relation which closes the standard [1] system of filtration equations for immiscible incompressible fluids. This system is composed of the equations of motion (generalized Darcy's law)

$$v = -kf(s)\partial p/\partial x,\ v_1 = -k_1 f_1(s)\partial p_1/\partial x \tag{0.2}$$

and the mass conservation law

$$m\partial s/\partial t + \partial v/\partial x = 0,\ -m\partial s/\partial t + \partial v_1/\partial x = 0. \tag{0.3}$$

Here, p is the pressure in the fluid (oil, for example), which occupies the fraction s of the pore space; $s_1 = 1 - s$ is the saturation of the second fluid (water), with a pressure p_1; k and k_1 are the total permeabilities of the medium, referred to the viscosities, at s = 1 and s = 0, respectively; f and f_1 are the relative ("phase") permeabilities; v and v_1 are the rates of filtration of the fluids; m is porosity; x is a coordinate; t is time.

The capillary pressure $p_c = p_c(s)$ is determined experimentally under static conditions and is assigned in the form of a fixed function of saturation s. In particular, the medium is assumed to be hydrophilic at $p_c \geq 0$ and hydrophobic at $p_c \leq 0$. However, the assumption that the sign of p_c in (0.1) is fixed is not always valid in problems of oil-field mechanics, since the problem of the wettability of the rocks which make up the oil-bearing strata cannot always be solved unambiguously [2, 3].

It was suggested in [4] that there are three main classes of porous media with regard to the case of two immiscible fluids saturating these media: 1) wettable ($p_c \geq 0$); 2) unwettable ($p_c \leq 0$); 3) intermediate-wettable or "mixed-wettable" [5]. A fluid–fluid–porous-medium system of the third type is characterized by a change in the sign of the function p_c.

As is known [6], the wettability of a system under static conditions is determined by the contact angle θ from the Young equilibrium equation

$$\cos\theta = (\gamma_1 - \gamma_2)/\gamma_{1,2}, \tag{0.4}$$

where γ_i (i = 1, 2) are the specific free energies of the interfaces between the skeleton and each of the fluids; $\gamma_{1,2}$ is the specific free energy of the interface between the fluids (surface tension). If $\gamma_1 > \gamma_2$, then the angle θ is acute, and fluid 2 wets the solid more readily than fluid 1. When $\gamma_1 \approx \gamma_2$, $\cos\theta \approx 0$, the fluids wet the solid equally well. The relationship between the values of γ_i may change over time even under static conditions

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